Enrollment No: _______________________ **Exam Seat No:**_______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name: Group Theory

Subject Code: 4SC05GTC1 **Branch: B.Sc. (Mathematics)**

Semester: 5 **Date: 23/03/2018** **Time: 10:30 To 01:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

a) Give an example of normal subgroup. **(01)**

b) Give an example of a non-commutative group. **(01)**

c) True or False: $(z_n, +_n)$ is a group. **(01)**

d) If $G = \{1, i, -1, -i\}$ is a group under multiplication then find $o(i)$. **(01)**

e) Define: Group **(02)**

f) Define: Isomorphism of group **(02)**

g) Find all the subgroups of $(Z_8, +_8)$. **(02)**

h) Prove that $o(a^p) \le o(a)$, $p \in Z$ for any element a in group G **(02)**

i) Is $(Z_7, +_7)$ cyclic group? If yes then find all generators of it. **(02)**

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions

a) Prove that the set $\{a + b\sqrt{2} \mid a, b \in Q; a^2 + b^2 \neq 0\}$ of R is a group under multiplication of two real numbers. **(05)**

b) For a subgroup H of G and for $a, b \in G$. Show that $Ha = Hb \Leftrightarrow ab^{-1} \in H$. **(05)**

c) Using Euler's theorem, find the remainder obtained on dividing 3^{256} by 14. **(04)**

Q-3 Attempt all questions

a) State and prove Lagrange's theorem. **(07)**

b) State and prove the first fundamental theorem of Homomorphism.`` **(07)**

Q-4 Attempt all questions

a) A subgroup of H of G is a normal subgroup of G iff $aHa^{-1} \subset H$, for $a \in G$ **(05)**

b) Prove that any two infinite cyclic groups are isomorphic. **(05)**

c) $H = \left\{ A \in GL(2;R) \middle| |A| \, is\, rational\, number \right\}$ is a subgroup under matrix multiplication of $GL(2;R)$. **(04)**

Q-5 Attempt all questions **(14)**

a) Let H is a subgroup of G iff $ab^{-1} \in H$ for $\forall a,b \in H$. **(05)**

b) The set $Z = \left\{ x \in G \middle| xy = yx, \, for \, \forall y \in G \right\}$ is a subgroup of G and also prove that if $a \in Z$ then $N(a) = G$. **(05)**

c) Check whether that $\{1,5,7,11\}$ is a subgroup of $\left(Z_{12}^{*}, \times_{12}\right)$ or not, where $\times_{12}$ is multiplication modulo 12. **(04)**

Q-6 Attempt all questions

a) Show that the set Q \ {-1} is an abelian group with respect to the binary operation $a*b = a+b+ab,$ for all $a,b \in G$. **(05)**

b) Obtain all generators, all subgroups of $\left(Z_{18}, +_{18}\right)$ and draw prepare lattice diagram. **(05)**

c) Let $\phi:(G,*) \to (G',\Delta)$ is a Homomorphism. If H is a normal subgroup of G then $\phi(H)$ is a normal subgroup of G' . **(04)**

Q-7 Attempt all questions

a) State and prove Euler's theorem. **(05)**

b) If K is a subgroup of G and H is a normal subgroup of G then prove that **(05)**
i) $K \cap H$ is a normal subgroup of K and
ii) KH is a subgroup of G.

c) Find order of f^2 and $f \circ g$, where $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 5 & 2 & 8 & 7 & 6 & 4 \end{pmatrix}$; $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 1 & 5 & 7 & 8 & 6 \end{pmatrix}$. **(04)**

Q-8 Attempt all questions

a) Prove that order of permutation is the least common multiple of the length of its disjoint cycles. **(07)**

b) A cyclic group G with generators $\langle a \rangle$ is finite iff there exist a positive integer k such that $a^k = e$. **(07)**